Exercises for the course
F7017T, Water turbines

1. Exercises on fluid mechanics (6)
2. Exercises on turbines (8)
3. Exercises on spirals and draft tubes (3)
4. Exercises similarities and scale-up formula (13)
5. Exercises cavitation (6)
6. Exercises turbine selection (9)
7. Exercises runner design (to come)
8. Exercises on unsteadiness (to come)

Dr. Michel Cervantes
Division of Fluid Mechanics
Luleå University of Technology
Tel: +46 (0)920 492143
E-mail: michel.cervantes@ltu.se
1 - Exercises on fluid mechanics

Exercise 1-1: (Momentum principle – circular jet– From Hydromekanik, H. Gustavsson)

A circular jet with diameter d and flow rate Q hit a disk normally. Determine the force on the disc when it
a) stands
b) moves in the jet direction with the velocity v
c) determine the maximal power you can transmit to the plate

Answer: a) \( F = \rho \frac{Q^2}{A} \), b) \( F = \rho \left( \frac{Q}{A} - v \right)^2 \), c) \( P_{\text{max}} = \frac{4}{27} \rho \frac{Q^3}{A^2} \)

Exercise 1-2: (Momentum principle – circular jet– From Hydromekanik, H. Gustavsson)

A fluid flows out a pipe through a small circular hole, see figure. Determine the force on the end plate.

Answer: \( F = \rho Q^2 \left( \frac{1}{D^2} - \frac{2}{d^2} + \frac{D^2}{d^2} \right) \)
Exercise 1-3: (Angular momentum principle – sprinkler – From Hydromekanik, H. Gustavsson)

A sprinkler rotates with a constant angular velocity, see figure. The flow rate is Q and each arm has an outlet area A.

(a) Determine the torque $T$ necessary to maintain the sprinkler at an angular velocity $\omega$ and determine the breaking power. For which $\omega$ is it max?

(b) If the sprinkler is torque free, what is the angular velocity?

Answer: a) $T = \rho Q \left( \frac{Q}{4A} r - \omega \left( r^2 + b^2 \right) \right) \\
\omega_{\text{max}} = \frac{Q}{8A} \frac{r}{(r^2 + b^2)}$

Exercise 1-4: (Bernoulli equation – Pitot tube)

A Pitot tube is an instrument allowing the measurement of a fluid velocity such as water, see figure below. The streamlines at the pressure taps $P_B$ and $P_B'$ have the same direction as the incoming flow in steady conditions. However, flows in water turbines are highly unsteady due to the movement of the runner blades, necessary to extract energy from the fluid. Determine the fluid velocity function of the pressure at the different taps and the fluid density for steady and unsteady conditions.

Answer: $U_{\text{steady}} = \sqrt{\frac{2\rho}{A}(P_A - P_B)}$, $U_{\text{unsteady}} = \frac{1}{N} \sum_{i=1}^{N} \sqrt{\frac{2\rho}{A} \left( P_{A_i} - \frac{P_{B_i} + P'_{B_i}}{2} \right)}$

Exercise 1-5: (Bernoulli equation – Gibson method)

Flow measurements in water turbines are crucial for efficiency measurements on full-scale machines. A good accuracy is difficult to obtain for low head machines due to the shortness of the water passages. Luleå University of Technology works on the development of the Gibson method for very low head. The method is based on the measurement of the differential pressure $\Delta P$ in the penstock during the closure of the guide vanes situated at see level $h$, see figure. However, guide vanes do not close completely; there is always some leakage, which has to be determined in order to calculate the flow rate with the Gibson method.

Assuming the penstock cross sections at the pressure taps 1 and 2 equal to $A$ and much larger than the guide vanes opening area $a$, determine the guide vanes leakage flow $q$ taking advantage of the pressure taps $P_1$ and $P_2$. $h_1$ and $h_2$ represent the pressure tape position above the guide vanes.

Answer: $q = a \left( \sqrt{h_1} + \sqrt{h_2} \right) \sqrt{\frac{g}{2}}$
Exercise 1-6: (Bernoulli equation – flow measurement – From Hydromekanik, H. Gustavsson)

The flow rate of a stream may be measured by placing an obstacle, see figure below. Assuming a negligible slop of the obstacle and water surface, a negligible velocity upstream of the obstacle and a nearly constant velocity over the obstacle, show that $d + h$ has a minimum function of the velocity for which $u = (gq)^{1/3}$, $d = \left(\frac{q^2}{g}\right)^{1/3}$ and $h = \frac{1}{2}\left(\frac{q^2}{g}\right)^{1/3}$.
2 - Exercises on turbines

Exercise 2.1: Pelton turbine (jet diameter – From Hydromekanik, H. Gustavsson)
A pipe 300 m long with a diameter of D_p=15 cm and a friction coefficient \( \lambda =0.02 \) supplies water from a reservoir situated 150 m above see level to a Pelton nozzle situated 90 m above see level. Assuming the losses in the nozzle of the form \( 0.04 \frac{V^2}{2g} \), determine the jet diameter \( D_j \), which gives maximal effect. Inlet losses from the reservoir to the pipe are neglected.

Answer: \( D_j = \left( \frac{1+\xi}{2\lambda L} D_p^2 \right)^{1/4}, D_j=5 \text{ cm} \)

Exercise 2.2: Pelton turbine (power, rotational speed, moment – From Hydromekanik, H. Gustavsson)
A Pelton turbine with a head \( H=670 \text{ m} \) is considered. The pipe between the reservoir and the turbine is \( L=6 \text{ km} \) long with a diameter \( D_p=1 \text{ m} \) and a friction coefficient \( \lambda =0.015 \). The water jet leaving the nozzle has a diameter \( D_j =18 \text{ cm} \) and the runner diameter is \( D=2R=1.5 \text{ m} \). The efficiency of the turbine is \( \eta=85 \% \).

Assuming the losses in the nozzle of the form \( 0.04 \frac{V^2}{2g} \), perfect velocity conditions between the water jet and the runner velocity, the water spreads at \( \beta=170^\circ \) from the runner blades and neglecting the losses on the runner blades, determine:

1. the power of the turbine
2. the runner angular frequency
3. the torque acting on the runner

Answer: 1 - \( P = \eta \left( 1 - \cos \beta \right) \frac{\pi D_j^2}{16} \left[ \frac{2gh}{1+\xi + \lambda \frac{D_j}{D_p}} \right]^{3/2} \), \( P=14.1 \text{ MW} \)

2 - \( \omega = \frac{1}{2R} \left[ \frac{2gh}{1+\xi + \lambda \frac{D_j}{D_p}} \right]^{1/2} \), \( \omega=36.5 \text{ rad/s} \)

3 - \( T=P/\omega, T=386 \text{ kNm} \)

Exercise 2.3: Pelton turbine (jet diameter and hydraulic efficiency – From Hydromekanik, H. Gustavsson)
A Pelton turbine with a head \( H=360 \text{ m} \) and an efficiency \( \eta \) of 85 % develops a power \( P=4500 \text{ kW} \). The runner is driven by two water jets at 400 rpm. The losses in the nozzle are 3 % and the friction coefficient of the blades is \( k=0.85 \). The water spreads at \( 170^\circ \) from the runner blades. The velocity ratio between the water jet and the runner velocity is \( r = R \omega / u_j = 0.46 \). Neglecting the losses in the pipe, determine:

(a) the water jet diameter

(b) the hydraulic efficiency of the turbine.

Answer: (a) \( A_j = \frac{1}{2 \eta \rho g H} \frac{1}{\varphi \sqrt{2gH}} \) where \( \frac{1-\varphi^2}{\varphi^2} = 0.03 \), \( D_j=54 \text{ mm} \)

(b) \( \eta_{\text{hydraulic}} = 2 \varphi^2 r (1-r)(1- k \cos \beta) \), \( \eta_{\text{hydraulic}}=88 \% \)
**Exercise 2.4:** Net head (From Fluid Mechanics, Franzini and Finnemore, 16.16)

In the test of a Francis turbine, a pressure of 140 kPa was measured at a point A at the flange at the entrance to a spiral turbine-case with a diameter $D_s=600$ mm. Neglecting the small velocity head in the tailrace, find the net head on the turbine if the flow rate was $Q=2.5$ m$^3$/s and the flange was $z_A=3$ m above the tailrace.

Answer: $h = \frac{P_A}{\rho g} + \left( \frac{4Q}{\pi D_s^2} \right)^2 \frac{1}{2g} + z_A$, $h=21.1$ m

**Exercise 2.5:** Reaction turbine (runner diameter and power– From Hydromekanik, H. Gustavsson)

A reaction turbine has its guide vanes at an angle of $30^\circ$ and the leading edge runner blade angles makes an angle of $120^\circ$ relative to the tangent. The turbine blade height at the inlet is $1/4$ of their diameter. The water does not have any tangential velocity at the outlet of the runner. The head is of 15 m and the rotational speed of the runner is 16.67 rotations per seconds. The hydraulic efficiency is of 88 % and the total efficiency of 85 %. Determine the turbine runner diameter at the inlet and the power developed.

Answer: $D = 2 \sqrt{\frac{gh}{\omega^2 \sin(\beta_i - \alpha_i) \cos(\alpha_i) \sin(\beta_i)}}$, $D=0.251$ m,

$P = \frac{8}{\pi} \eta \rho \omega g D^3 \frac{\sin(\beta_i) \sin(\alpha_i)}{\sin(\beta_i - \alpha_i)}$, $P=35.2$ kW

**Exercise 2.6:** Power (From Fluid Mechanics, Franzini and Finnemore, 16.17)

A reaction turbine is supplied with water through a 1500 mm diameter pipe ($\lambda=0.011$ mm) that is 50 m long. The water surface in the reservoir is 27 m above the draft-tube inlet that is 4.1 m above the water level in the tailrace. If the turbine efficiency is 92% and the discharge is 12 m$^3$/s, what is the power output of the turbine in kilowatts? By how much much the discharge be increased to increase power production by 500 kW assuming $\lambda$ unchanged? Assume the velocity in the tailrace can be neglected.

Answer: $P = \eta \rho g Q \left( z_{tw} - \frac{1}{2g} \left( \frac{4Q}{\pi D^2} \right)^2 \frac{L}{D} \right)$, $P=3.3$ MW

$P + \Delta P = \eta \rho g (Q + \Delta Q) \left( z_{tw} - \frac{1}{2g} \left( \frac{4(Q + \Delta Q)}{\pi D^2} \right)^2 \frac{L}{D} \right)$, $\Delta Q = 1.938$ m$^3$/s

**Exercise 2.7:** Francis turbine (guide vanes– From Hydromekanik, H. Gustavsson)

A Francis turbine with a flow rate $Q=4$ m$^3$/s and a rotational speed $N=600$ rpm is considered. The runner has an outer diameter $D_1=1.2$ m, a blade angle leading edge $\beta=110^\circ$ and a blade height $b_1=0.1$ m. Determine the angle $\alpha$ of the guide vanes for an attached flow.

Answer: $\alpha = \cot(\beta_i) + \frac{\pi^2 D_1^2 b_1 N}{Q}$, $\alpha=17.4^\circ$

**Exercise 2.8:** Reaction turbine (efficiency and blade angle– From Hydromekanik, H. Gustavsson)

A reaction turbine has its guide vanes positioned at an angle $\gamma$. The runner blades make an angle of 90$^\circ$ to the tangential direction. The radial velocity trough the turbine is constant. Show that the maximal hydraulic efficiency is $\eta_h= 1/(1+0.5 \tan^2 \gamma)$. 

Last modified 2008-12-04 by Michel Cervantes - 6/19 -
3 - Exercises on spirals and draft tubes

Exercise 3.1: Spiral inlet boundary conditions (Optional!)

a) The general form for the development of the mechanical energy \( E \) assuming incompressible flow is obtained by multiplication of the Navier-Stokes equation with the velocity vector \( \vec{U} (u, v, w) \). Show that:

\[
\frac{DE}{Dt} = \frac{1}{\rho} \frac{\partial P}{\partial t} + \nu \vec{U} \nabla^2 \vec{U}, \quad \text{where} \quad E = \frac{P}{\rho} + \frac{1}{2} U^2.
\]

b) By taking the divergence of the Navier-Stokes equation and using the definition of the vorticity \( \omega \) and dissipation function \( \phi \), show that:

\[
\nabla^2 P = \frac{1}{2} \left( \rho \omega^2 - \frac{\phi}{\nu} \right), \quad \text{where} \quad \frac{\phi}{\mu} = \frac{\partial U_i}{\partial x_j} \frac{\partial U_j}{\partial x_i} + \frac{\partial U_j}{\partial x_j} \frac{\partial U_i}{\partial x_i} \quad \text{and} \quad \omega^2 = \frac{\partial U_i}{\partial x_j} \frac{\partial U_j}{\partial x_i} - \frac{\partial U_j}{\partial x_i} \frac{\partial U_i}{\partial x_j}
\]

and thus

\[
\frac{DE}{Dt} = \frac{1}{\rho} \frac{\partial P}{\partial t} + \nabla^2 E - \nu \omega^2
\]

What does \( \frac{1}{\rho} \frac{\partial P}{\partial t} \) and \( \nu \omega^2 \) represent?

c) Show that the convective terms of the Navier-Stokes may be written \( \vec{U} \nabla \vec{U} = \nabla \left( \frac{U^2}{2} \right) - \vec{U} \times \vec{\omega} \).

By applying the divergence operator to the preceding relation, show that: \( \nabla^2 E = \text{div}(\vec{U} \times \vec{w}). \)

d) Volume integrate \( \nabla^2 E \) over a spiral and show that:

\[
\iiint \nabla^2 E dV = \iint \text{div}(\vec{U} \times \vec{w}) dS
\]

What is the contribution of \( \nu \nabla^2 E \) to the variation of the mechanical energy in the spiral function of the type of flow, i.e. with or without secondary flow?

Exercise 3.2: Draft tube (From Fluid Mechanics, Franzini and Finnemore, 16.18, modified)

A draft tube leading from the discharge side of a turbine to the submerged discharge in the tailrace consists of a pipe \( \lambda = 0.025 \) of constant diameter 0.8 m and length 10.0 m. The flow rate is 9.2 m\(^3\)/s. If this draft tube was to be replaced by a diverging tube 12.0 m long whose diameter increase linearly from \( D_0 = 0.8 \) m to \( D_1 = 2.5 \) m over the 12.0 m length and an identical surface roughness as the precedent draft tube, how much additional head would be developed by the replacement draft tube? Both draft tube will be considered straight without bend.

Answer:

a) Straight tube draft tube

\[
h_{\text{friction}} = \frac{1}{2g} \left( \frac{4Q}{\pi D^2} \right)^2 \frac{L}{D} \lambda \quad \text{and} \quad h_{\text{discharge}} = \frac{1}{2g} \left( \frac{4Q}{\pi D^2} \right)^2
\]

\[h = 5.3 + 17.1 = 22.4 \text{ m}\]
Exercise 3.3: Draft tube (From Fluid Mechanics, Franzini and Finnemore, 16.19)

At its upper end a draft tube has a diameter of 24.5 in where it joins the discharge side of the turbine at a point 11.0 ft above the surface of the water in the tailrace. The discharge end of the draft tube has a diameter of 42 in and the velocity in the tailrace is negligible. The total head loss in the draft tube is $0.15V_2^2/2g$ plus the submerged discharge loss of $V_3^2/2g$, where subscripts 2 and 3 refer to the upper and lower ends of the draft tube, respectively,

(a) When the flow is 38.8 cfs, what is the pressure at the upper end of the draft tube?
(b) Suppose the draft tube was of uniform diameter; what then would be the pressure at the upper end of the tube?
(c) How much head is saved by the diverging tube? Assume the draft tube has a length of 18 ft and $\lambda = 0.020$.

Answer:

(a) $P_2 = \frac{1}{2} \rho \left( -0.85 \left( \frac{4Q}{\pi D_2^2} \right)^2 + 2 \left( \frac{4Q}{\pi D_3^2} \right)^2 - 2gz_2 \right)$

$P_2 = -3.73 \times 10^4$ Pa

(b) $P_2 = \frac{1}{2} \rho \left( 1.15 \left( \frac{4Q}{\pi D_2^2} \right)^2 - 2gz_2 \right)$

$P_2 = -2.47 \times 10^4$ Pa

(c) $h = 1.26$ m
4 - Exercises on similarities and scale-up

Exercise 4.1: Scale-up (From Fluid Mechanics, Franzini and Finnemore, 16.20)
A model turbine, one-twentieth of prototype size, has a maximum hydraulic efficiency of 86.2%. Estimate the efficiency of the prototype utilizing the Moody step-up formula.

Answer:
\[
\frac{1-\eta_M}{1-\eta_P} = \left( \frac{D_P}{D_M} \right)^{\frac{1}{3}}, \quad \eta_P = 92.4 \%
\]

Exercise 4.2: Dimensionless numbers (From Fluid Mechanics, Franzini and Finnemore, 16.21)
A turbine runs at 150 rpm, discharges 200 cfs and develops 1600 bhp under a net head of 81 ft.

(a) What is its efficiency?

(b) What would be the revolutions per minute, \( Q \), and brake horsepower of the same turbine under a net head of 162 ft for homologous conditions?

Answer:
(a) \( \eta = \frac{P}{\rho g Q H} \), \( \eta = 85.7 \%
\]

(b) \( n_{11} = \frac{nD}{\sqrt{H}} = cst \), \( n=212 \) rpm \( Q_{11} = \frac{Q}{D^2 \sqrt{H}} = cst \), \( Q=8 \) m\(^3\)/s

\[
P_{11} = \frac{P}{D^2 H \sqrt{H}} = cst \), \( P=4525 \) bhp

Exercise 4.3: Dimensionless numbers (From Fluid Mechanics, Franzini and Finnemore, 16.22)
If a turbine homologous to that of Prob. 16.21 has a runner of twice the diameter, what would be the revolutions per minute, \( Q \), and brake horsepower under the same head of 81 ft?

Answer:
\( n_{11} = \frac{nD}{\sqrt{H}} = cst \), \( n=75 \) rpm

\( Q_{11} = \frac{Q}{D^2 \sqrt{H}} = cst \), \( Q=22.64 \) m\(^3\)/s

\[
P_{11} = \frac{P}{D^2 H \sqrt{H}} = cst \), \( P=6400 \) bhp

Exercise 4.4: Model test (From Fluid Mechanics, Franzini and Finnemore, 16.23)
A 2.6 m diameter reaction turbine is to be operated at 200 rpm under a net head of 30 m. A 1:10 model of this turbine is built and tested in the laboratory. If the model is operated at 600 rpm, under what net head should it be tested to simulate normal operating conditions in the prototype?

Answer:
\( n_{11} = \frac{nD}{\sqrt{H}} = cst \), \( H=2.7 \) m
Exercise 4.5: Dimensionless numbers (From Fluid Mechanics, Franzini and Finnemore, 16.24)
A small Francis turbine ($N_s=30, \ D=2\ ft$) is tested and found to have an efficiency of 0.893 when operating under optimum conditions. Approximately what would be the maximum efficiency of a homologous runner ($N_s=30$) with a diameter of 6 ft.

Answer:

$$1 - \eta _{M} \left( \frac{D_p}{D_M} \right)^{1/3}, \ \eta _{p} = 91.4\%$$

Exercise 4.6: Model test (From Fluid Mechanics, Franzini and Finnemore, 16.25)
A 12-ft-diameter reaction turbine is to be operated at 100 rpm under a net head of 96ft. A 1:8 model of this turbine is built and tested in the laboratory. If the model is operated at 450 rpm, under what net head should it be tested to simulate normal operating conditions?

Answer: $n_{11} = \frac{nD}{\sqrt{H}} = \text{cst}, \ H_M = 9.3\ m$

Exercise 4.7: Model to prototype (From Fluid Mechanics, Franzini and Finnemore, 16.26)
A 1:8 model of a 12 ft diameter turbine is operated at 600 rpm under a net head of 54.0 ft. Under this mode of operation the bhp and $Q$ of the model were observed to be 332 hp and 62 cfs, respectively. From the above data compute

(a) the specific speed of the model and the value of $\phi$

(b) the efficiency and shaft torque of the model

(c) the efficiency of the prototype

(d) the flow rate and horsepower of the prototype if it is operated at 450 rpm under a net head of 200 ft

Answer:

(a) $N_s = \frac{n\sqrt{P}}{H^4}, \ N_s = 330 \quad \text{with} \ H(m), \ P(bhp) \ \text{and} \ n(rpm)$

$$\phi = \frac{u_i}{\sqrt{2gH}} = \frac{\pi n_M D_M}{\sqrt{2gH}}, \ \phi = 0.79 \quad (D_M = 1/8 \ D_p = 12/8 \ ft)$$

(b) $\eta_{M} = \frac{P_M}{\rho gQ_M H_M}, \ \eta _{M} = 86\%$

$$T = \frac{P}{\omega}, \ T = 3.9\ kNm$$

(c) $1 - \eta _{M} \left( \frac{D_p}{D_M} \right)^{1/3}, \ \eta _{p} = 90.8\%$

(d) $n_{11} = \frac{n_p D_p}{\sqrt{H_p}} = \frac{450 \times 12 \times 8}{\sqrt{200}} = 3054.7 \quad \text{and} \ n_{11M} = \frac{n_M D_M}{\sqrt{H_M}} = \frac{600 \times 12}{\sqrt{54}} = 979.8$
Exercise 4.8: Dimensionless numbers (From Fluid Mechanics, Franzini and Finnemore, 16.27)

Calculate the specific speed of a Pelton turbine with the following characteristics: static head=2418 ft, net head=2200 ft, n=250 rpm, pitch diameter=162 in. Estimate the runner diameter with the help of the figure below, BG units are used.

Answer:

For Pelton turbine, \( \phi = 0.5 \), since \( \phi = \sqrt[3]{\frac{n^2 \cdot \sqrt{bhp}}{gH}} \) we get \( D = \frac{153.3 \cdot \phi \cdot \sqrt{H}}{n} \), \( D = 14.4 \) ft

Exercise 4.9.: Dimensionless numbers (From Fluid Mechanics, Franzini and Finnemore, 16.28)

Find the specific speed of a Francis turbine with the following characteristics: \( P = 128,000 \) kW, \( H = 87.6 \) m and \( n = 150 \) rpm. Estimate the runner diameter.

Answer:

\[
N_S = \frac{n \sqrt{P}}{H^{3/4}}, \quad N_S = 52.9
\]

\( \phi = 0.7 \), \( D = \frac{153.3 \cdot \phi \cdot \sqrt{H}}{n} \), \( D = 12.10 \) ft

Exercise 4.10: Dimensionless numbers (From Fluid Mechanics, Franzini and Finnemore, 16.31)

Find the specific speed of a turbine that runs at a maximum efficiency of 90% at 300 rpm under a net head of 81 ft with a flow rate of 50 cfs. Estimate the runner diameter.
**Exercise 4.11:** Dimensionless numbers (From Fluid Mechanics, Franzini and Finnemore, 16.32)

A double-overhung impulse turbine is considered. Each wheel of the two-wheel unit develops 25,000 hp at 500 rpm under a head of 5330 ft.

(a) What is the specific speed of these wheels?

(b) Estimate their diameter and compare your answer with their actual diameter of 10.89 ft.

**Exercise 4.12:** Dimensionless numbers (From Fluid Mechanics, Franzini and Finnemore, 16.34)

A Kaplan turbine at Rock River, Illinois develops 800 hp at 80 rpm under a head of 7 ft.

(a) What is the specific speed of this turbine?

(b) Estimate the runner diameter and compare your answer with the actual diameter of 136 in.

**Exercise 4.13:** Francis turbine (dimensionless numbers– From Hydromekanik, H. Gustavsson)

A Francis prototype will have a specific speed $n_s=210$ and develop a power of 30 MW at 180 rpm. A model of the Francis prototype will be run with $Q=0.6$ m$^3$/s with a head of 4.5 m. Assuming an efficiency of 88 %, determine the model rotational speed, power and dimension relative to the prototype.

Answer: $N_m=285$ rpm, $P_m=\rho g Q_m H_m \eta$, $D_p=5.5 D_m$, $H_p=63.6$ m
5 - Exercises on cavitation

**Exercise 5.1:** Cavitation (From Fluid Mechanics, Franzini and Finnemore, 16.35)
Consider the case of a Francis turbine having a specific speed of 40 that is set 10 ft above tailwater elevation. Perform calculations using Fig. 16.17 and Eq. (16.22) to find the maximum permissible head under which this turbine should operate in order to be safe from cavitation. Check your calculated result against the information shown on Fig. 16.16. Do they agree?

Answer:
Using Franzini and Finnemore

From figure 16.17, \( \sigma = 0.14 \)

From equation 16.23, \( H \leq \frac{1}{\sigma} \left( \frac{P_{\text{atm}}}{\rho g} - \frac{P_{\text{w,v}}}{\rho g} - H_s \right) \)

Using Krivchenko:

Recalculation of \( N_s \):
\[ N_{s_{\text{Krivchenko}}} = N_{s_{\text{Franzini}}} \left( \frac{1}{0.3048} \right)^{\frac{5}{4}} = 176.4 \]

With equation 7.15:
\[ \sigma = \left( \frac{N_s + 30}{200000} \right)^{1.8} = 0.0734 \]

With equation 7.13:
\[ H \leq \frac{1}{\sigma} \left( \frac{P_{\text{atm}}}{\rho g} - \frac{P_{\text{w,v}}}{\rho g} - H_s \right), H \leq 97.3 \text{ m} \]

**Exercise 5.2:** Cavitation (From Fluid Mechanics, Franzini and Finnemore, 16.36)
Consider the case of a propeller turbine having a specific speed of 150 that is set 5 ft below tailwater. Perform calculations using Fig. 16.17 and Eq. (16.22) to find the maximum permissible head under which this turbine should operate in order to be safe from cavitation. Check your calculated result against the information shown in Fig. 16.16. Do they agree?

Answer:
Using Krivchenko,
\[ N_{s_{\text{Krivchenko}}} = N_{s_{\text{Franzini}}} \left( \frac{1}{0.3048} \right)^{\frac{5}{4}} = 661.5, \ \sigma = 0.6465 \text{ and } H \leq 17.8 \text{ m} \]
Exercise 5.3: Cavitation (From Fluid Mechanics, Franzini and Finnemore, 16.37)

At its maximum efficiency of 93% a turbine delivers 3000 hp to the shaft under a head of 72 ft when operating at 300 rpm. Find the following:

(a) the flow rate through the turbine
(b) the specific speed of the turbine
(c) the approximate diameter of the turbine runner
(d) the elevation at which the turbine should be set to be safe against cavitation if it is installed at sea level.

Answer:

(a) \( \eta = \frac{P}{\rho g Q H} \), \( Q = 10.98 \, \text{m}^3/\text{s} \)

(b) \( N_S = \frac{n \sqrt{P}}{H^{1.4}} \), \( N_S = 78.3 \) with \( H(\text{ft}) \), \( P(\text{bhp}) \), \( n(\text{rpm}) \)

(c) \( \phi = 0.7 \), \( D = \frac{153.3}{n} \sqrt{H} \), \( D = 3 \, \text{ft} \)

(d) Using Krivchenko:

Recalculation of \( N_s \): \( N_{S,\text{k rivchenko}} = N_{S,\text{Franzini}} \left( \frac{1}{0.3048} \right)^{\frac{5}{3}} = 345.9 \)

With equation 7.15: \( \sigma = \left( \frac{N_S + 30}{200000} \right)^{1.8} = 0.216 \)

With equation 7.13: \( H_S \leq \frac{P_{\text{am}}}{\rho g} - \frac{P_{\text{tw}}}{\rho g} - \sigma H \), \( H_S \leq 5.2 \, \text{m} \)

Exercise 5.4: Cavitation (From Fluid Mechanics, Franzini and Finnemore, 16.38)

A propeller turbine operates at a maximum efficiency of 92% under a head of 30 ft at 450 rpm and develops a shaft power of 620 hp. Find

(a) the flow rate through the turbine
(b) the specific speed of the turbine
(c) the approximate diameter of the turbine runner
(d) how far above tailwater elevation the turbine can be set and still be safe from cavitation. Assume the turbine is at an elevation of 100 ft.
Exercise 5.5: Cavitation (From Fluid Mechanics, Franzini and Finnemore, 16.39)
Repeat part (d) of Prob. 16.38 for the case where the turbine is installed at elevation 5000 ft.

Exercise 5.6: Cavitation (From Fluid Mechanics, Franzini and Finnemore, 16.40)
A turbine whose specific speed is 80 is to operate under a head of 50 ft at an elevation where the atmospheric pressure is 12.8 psia. The water temperature is 50°F.

(a) If this turbine is set 6 ft above tailwater, will it be safe from cavitation?

(b) What is the highest permissible elevation of this turbine with respect to tailwater when operating under a head of 60 ft?
6 - Exercises on turbine selection

Exercise 6.1: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.41)

(a) A turbine is to be installed at a point where the available head is 175 ft and the available flow will average 1000 cfs. What type of turbine would you recommend? Specify the operating speed and the number of generator poles for 60-cycle electricity if a turbine with the highest tolerable specific speed to safeguard against cavitation is selected. Assume a draft head of 10 ft and 90% turbine efficiency. Approximately what size of turbine runner is required?

(b) For the same conditions, select a set of two identical turbines to be operated in parallel. Specify the speed and size of the units.

Answer:

(a) Safe from cavitation: \[ \sigma \leq \frac{1}{H} \left( \frac{P_{\text{in}}}{\rho g} - \frac{P_{\text{w}}}{\rho g} - H_s \right), \quad \sigma \leq 0.13 \]

With equation 7.15: \[ \sigma = \left( \frac{N_S + 30}{200000} \right)^{1.8}, \quad N_S \leq 253.6 \rightarrow \text{Francis turbine} \]

Figure 5.16 in Krivchenko, \( n_\text{i} \leq 75 \text{ rpm and } Q_\text{i} \leq 0.1 \text{ m}^3/\text{s} \)

\[ \rightarrow \quad n \leq 273 \text{ rpm and } D \leq 2 \text{ m} \]

Number of poles is \( P=28 \rightarrow n=257 \) (see Krivchenko page 172)

Exercise 6.2: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.41)

What is the least number of identical turbines that can be used at a powerhouse where the available head is 1200ft and \( Q = 1650 \text{ cfs} \)? Assume turbine efficiency is 90% and speed of operation is 138.5 rpm. Specify the size and specific speed of the units.

Exercise 6.3: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.46)

It is desired to develop 15,000 bhp under a head of 1000 ft. Make any necessary assumptions and estimate the diameter of the wheel required and the rotative speed.

Exercise 6.4: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.47)

A single hydraulic turbine is to be selected for a power site with a net head of 100 ft. The turbine is to produce 25,000 hp at maximum efficiency. What speed (rpm) and diameter should this turbine have if

(a) a Francis turbine is selected

(b) a propeller turbine is selected?

(c) What are the highest "settings" (above or below tailwater) that should be recommended for each of these machines for them to run cavitation-free at their points of maximum efficiency?
Exercise 6.5: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.48)
For 50-cycle electricity how many poles would you recommend for a generator that is connected to a turbine operating under a design head of 3000 ft with a flow of 80 cfs? Assume turbine efficiencies as given in Fig. 16.14 and be sure the turbine is free of cavitation.

Exercise 6.6: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.49)
Specify the type, speed, and size of a single turbine to be installed at a site with an effective head of 48 m, a maximum draft head of 2 m, and a flow rate of 5 m³/s. How would your recommendation change if the available flow was 50 m³/s?

Exercise 6.7: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.50)
It is desired to develop 300,000 hp under a head of 49 ft and to operate at 60 rpm.
(a) If turbines with a specific speed of approximately 150 are to be used, how many units would be required?
(b) If Francis turbines with a specific speed of 80 were to be used, how many units would be required?

Exercise 6.8: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.51)
Select two, four, and six identical turbines for an installation where \( h = 400 \) ft and total \( Q = 300 \) cfs. Develop 60-cycle electricity using either 36- or 72-pole generators. Be sure your selection is free of cavitation. Assume the turbine efficiency is 90%.

Exercise 6.9: Turbine selection (From Fluid Mechanics, Franzini and Finnemore, 16.52)
A turbine is to be installed at a point where the net available head is 35 m and the available flow will average 23 m³/s. What type of turbine would you recommend? Specify the operating speed and number of generator poles for 60-cycle electricity if a turbine with the highest tolerable specific speed to safeguard against cavitation is selected. Assume a draft head of 3 m and 90% turbine efficiency. Approximately what size of turbine runner is required?
7 - Exercises on runner design
8 - Exercises on unsteadiness

**Exercise 8.1:** Oscillating plate (From Advanced Fluid Mechanics, H. Gustavsson, 4.12)

The space above an infinite plate is filled with a liquid. The plate oscillates in its plane with angular frequency $\omega$. Use the velocity distribution over one period of the motion $(2\pi/\omega)$ per $m^2$, the following quantities:

(a) the dissipation integrated the y-direction

(b) the total kinetic energy

(c) the power needed to drive the plate