Modeling of moisture transport in wood below the fiber saturation point

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Schedule

• Basics of moisture transport in wood
  – Physical background
  – Formulation

• Homogenization of material properties
  – Thermal conduction, validation
  – Steady state diffusion, validation

• Modeling of transient transport processes
  – Implementation in FE-code

• Summary, outlook
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Processes on the microscale

Macroscopic description
Static conditions

Steady state conditions
**Transient conditions**

![Diagram showing transient conditions in wood drying](image)

- Lumen
- Cell wall
- Bound water
- Macroscopic disequilibrium

**Sorption isotherms**

![Graph of sorption isotherms](image)

- Equilibrium moisture content $\theta_e$
- Water vapor concentration $c_v$ [g/m$^3$]

11th International IUFRO Wood Drying Conference 2010, Skellefteå
Formulation of transient transport processes

- 3 coupled macroscopic differential equations
  - Mass conservation for bound water
    \[ \int_V \frac{dc_b}{dt} \, dV + \int_{S_{ch}} \mathbf{n} \cdot \mathbf{J}_b \, dS + \int_V \dot{\varepsilon} \, dV = 0 \]
  - Mass conservation for water vapor
    \[ \int_V \frac{dc_v}{dt} \, dV + \int_{S_{sv}} \mathbf{n} \cdot \mathbf{J}_v \, dS + \int_V \dot{\varepsilon} \, dV = 0 \]
  - Energy conservation
    \[ \int_V \rho_c \dot{\varepsilon} \, dV + \int_{S_T} \mathbf{n} \cdot \mathbf{f} \, dS + \int_{S_{th}} h_b \mathbf{n} \cdot \mathbf{J}_b \, dS + \int_{S_{sv}} (h_v - h_w) \mathbf{n} \cdot \mathbf{J}_v \, dS + \int_V (h_v - h_b) \dot{\varepsilon} \, dV = 0 \]

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Transport properties – homogenization scheme

Method:
Continuum micromechanics

Thermal conduction - results

* spruce
* pine
* larch
* balsa
* oak
* beech

T = ~ 27 °C
u = ~ 12 %
Steady state diffusion - validation

Spruce
0.404 g/cm³

radial

- 100 °C
- 80 °C
- 60 °C
- 40 °C

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Formulation of transient transport processes

• 3 coupled macroscopic differential equations

\[
\begin{align*}
\int \frac{d\rho}{dt} \, dV + \int \mathbf{n} \cdot \mathbf{J} \, dS + \int \hat{c} \, dV &= 0 \\
\int \frac{d\tau}{dt} \, dV + \int \mathbf{n} \cdot \mathbf{J} \, dS + \int \hat{c} \, dV &= 0 \\
\int \frac{dm_{H_2O}}{dt} \, dV + \int \mathbf{n} \cdot \mathbf{J} \, dS + \int (h_v - h_w) \mathbf{n} \cdot \mathbf{J} \, dS + \int (h_v - h_w) \hat{c} \, dV &= 0
\end{align*}
\]

→ finite element model

• sub-model of cell wall for the coupling term

→ one-dimensional finite difference model in each integration point of the finite element mode

Sub-model for the cell wall

[Diagram showing a sub-model for the cell wall with notation and labels for symmetry boundary condition and half cell wall thickness.]
Summary

• Physical background
  – Microscale processes – macroscopic description
  – Differences between steady state and transient transport processes

• Homogenization of material properties
  – Starting from sample-independent properties
  – Consideration of morphology and composition
  – Applicable to any wood species

• Modeling of transient transport processes
  – 3 solution variables, sub-model for cell wall
  – Finite element method
Thank you very much for your attention!